

Comments on “Searching for a continuum limit in CDT quantum gravity”

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Abstract

To facilitate the search for a continuum limit of causal dynamical triangulations, Ambjørn, Coumbe, Gizbert-Studnicki, and Jurkiewicz recently reported measurements of the lattice spacing as a function of the bare couplings. Although these authors’ methods are technically sound, the conclusions that they draw from their analyses rest crucially on certain unstated assumptions. I elucidate these assumptions, and I argue that our current understanding of causal dynamical triangulations does not entail their justification.

Causal dynamical triangulations is an approach to the quantization of classical theories of gravity based on a particular lattice regularization of the associated path integral. (See [9, 18] for reviews.) A key question facing such an approach is that of the existence of a continuum limit in which physical quantities remain finite while the lattice regularization is removed. (The physical nature of this continuum limit constitutes a further—contingent—key question.) Practitioners of causal dynamical triangulations hope that a continuum limit is realized through the presence of a non-Gaussian ultraviolet fixed point, confirming Weinberg’s asymptotic safety conjecture [24].

To determine if such a fixed point exists for a lattice-regularized theory, one first searches for second (or higher) order transitions within its phase structure. The ground state of the causal dynamical triangulations of 4-dimensional Einstein gravity (for 3-sphere spatial topology) possesses a rich phase structure with multiple phase boundaries at which higher order transitions exist [2, 11, 12, 21]. Moreover, within its so-labeled phase C, which abuts most of these higher order transitions, the ground state exhibits physical semiclassical properties on sufficiently large scales [4, 5, 7, 8, 9, 13, 14, 10, 23]. Having located second (or higher) order transitions, one next performs a renormalization group analysis. If an ultraviolet fixed point exists along any of these transitions, then there exist renormalization group trajectories flowing into this fixed point along which the lattice regularization necessarily vanishes. Drawing on the phenomenology of phase C, Ambjørn, Görlich, Kreienbuehl, Jurkiewicz, and Loll made the first attempts to locate an ultraviolet fixed point and es-

tablish a continuum limit [6]. Cooperman subsequently proposed a related but distinct renormalization group scheme [19]. In the recent paper “Searching for a continuum limit in CDT quantum gravity”, Ambjørn, Coumbe, Gizbert-Studnicki, and Jurkiewicz (ACGSJ) seek to continue this search [3].

ACGSJ aim to determine the dependence of the lattice spacing on the bare couplings within phase C. They foresee this information guiding the search for a continuum limit as discussed in the previous paragraph: if there exists an ultraviolet fixed point within the ground state’s phase structure, then the lattice spacing approaches zero along renormalization group trajectories flowing into this fixed point. To accomplish their aim, ACGSJ employ two methods, both based on established phenomenology of phase C, to ascertain the value of the lattice spacing. First, they measure and model the amplitude of perturbations in the spatial 3-volume [8, 19]; second, they measure and scale the diffusion time dependence of the spectral dimension [15, 16, 22, 23]. Although the techniques that they employ in these two analyses are well-founded, I question the conclusions that they draw on the basis of their analyses. Specifically, I show that their analyses’ conclusions rest crucially on respective unstated assumptions. While these assumptions could possibly be (approximately) justified, I argue that currently they are not. I now critique their two analyses in turn. I preface each critique with an observation that lays bare the respective assumption [20].

The lattice spacing is dimensionful. Since one can only measure dimensionless quantities—a fact underscored by computer simulations—one cannot measure

the lattice spacing itself, but one can measure the ratio of the lattice spacing to another length scale. More typically, whenever one measures a length, one actually measures the ratio of that length to an established standard unit of length. Comparing two different lengths requires that one measure both lengths with respect to the same unit of length.

In their first analysis ACGSJ seem to lose sight of this last fact. Employing a technique first demonstrated in [8]—modeling perturbations in the numerically measured spatial 3-volume about its ensemble average as linear gravitational perturbations propagating on Euclidean de Sitter space—they obtain an expression, their equation (9), for the lattice spacing \mathfrak{a} in units of the Planck length ℓ_P . This technique thus yields a value for the dimensionless ratio \mathfrak{a}/ℓ_P , as the previous paragraph’s observation demands. They apply this technique to two sets of ensembles of causal triangulations: the first set consisting of four ensembles characterized by different values of κ_0 but the same value of Δ , the second set consisting of five ensembles characterized by the same value of κ_0 but different values of Δ . (κ_0 and Δ are the two bare couplings.) They analyze their results by directly comparing values of \mathfrak{a}/ℓ_P across the ensembles within each set. They claim to compare values of \mathfrak{a} , but, of course, they do not—and cannot—perform such a comparison.

As ACGSJ have actually measured \mathfrak{a}/ℓ_P , they can only meaningfully compare values of \mathfrak{a} for different values of the bare couplings by assuming that ℓ_P has the same value for different values of the bare couplings. They neither acknowledge this fact nor invoke this assumption, so their comparison of values of \mathfrak{a}/ℓ_P remains unjustified.

What justification could ACGSJ offer for the equivalence of ℓ_P for different values of the bare couplings? Recall first that ℓ_P is proportional to the square root of the renormalized Newton constant G ; accordingly, equivalence of ℓ_P for different values of the bare couplings follows from equivalence of G for different values of the bare couplings. Now, ignoring momentarily the difficulty of making precise the renormalization group flow of a dimensionful coupling [1], suppose that G experiences a nontrivial renormalization group flow. (If its flow is trivial, then this paragraph’s opening question is also trivial.) For G to have the same value for different values of the bare couplings, ACGSJ’s numerical measurements must probe G at the same scale for different values of the bare couplings. At what scale do they probe G ? They essentially measure just three quantities to determine the value of \mathfrak{a}/ℓ_P : the spacetime 4-volume, the spatial 3-volume as a function of time, and the covari-

ance of perturbations in the spatial 3-volume as functions of time. The spacetime 4-volume yields the largest scale characterizing the ground state, which ACGSJ model as the de Sitter length ℓ_{dS} . The spatial 3-volume and the covariance of perturbations in the spatial 3-volume vary on scales between $O(1)\ell_{\text{dS}}$ and $O(10^{-1})\ell_{\text{dS}}$ [19]. If G is approximately constant on these (relatively) large scales, then G is approximately equivalent for different values of the bare couplings, assuming that ℓ_{dS} has the same value for different values of the bare couplings.

Attempting to justify the assumption of equivalence of ℓ_P for different values of the bare couplings has led to the assumption of equivalence of ℓ_{dS} for different values of the bare couplings. At some point this chain of assumptions terminates with the establishment of a standard unit of length. By definition, a standard unit of length has a fixed length, the constancy of which we judge to be consistent with our current scientific knowledge. For instance, metrologists have continually updated the definition of the second to reflect our continually advancing understanding of the universe [17]. ACGSJ could choose ℓ_P as a standard unit of length, thereby justifying their analysis. They would then be obligated to demonstrate this choice’s consistency with our current understanding of the ground state’s phenomenology. Alternatively, ACGSJ could choose ℓ_{dS} as a standard unit of length in which case they would additionally be obligated to demonstrate that all of the ensembles considered have the same value of ℓ_P in units of ℓ_{dS} . Cooperman further discusses these and several other possibilities for a standard unit of length [19].

The preceding discussion, particularly that of the previous paragraph, illuminates the supposition that G exhibits a nontrivial renormalization group flow. Since one cannot measure dimensionful quantities, one must consider the renormalization group flow of the ratio of a dimensionful coupling to another quantity of the same dimensions. For instance, one could consider the renormalization group flow of ℓ_P/ℓ_{dS} , but one could attribute changes in ℓ_P/ℓ_{dS} neither to ℓ_P nor to ℓ_{dS} alone; however, if one could establish ℓ_{dS} as a standard unit of length, then one could attribute changes in ℓ_P/ℓ_{dS} to ℓ_P alone. In light of these considerations, ACGSJ must carefully examine the consistency of their comparison of values of \mathfrak{a}/ℓ_P for different values of the bare couplings.

A renormalization group trajectory is a sequence of theories all of which describe the same physics each over a different interval of scales. For instance, along a renormalization group trajectory generated by successive coarse grainings, the interval of scales evolves from $(\ell_{\text{UV}}, \ell_{\text{IR}})$ to $(\ell_{\text{UV}} + \delta\ell, \ell_{\text{IR}})$ to $(\ell_{\text{UV}} + 2\delta\ell, \ell_{\text{IR}})$

et cetera; the ultraviolet scale ℓ_{UV} successively increases while the infrared scale ℓ_{IR} remains constant. This perspective on renormalization group trajectories has the following immediate consequence: if one computes a scale-dependent physical observable $\mathcal{O}(\ell)$ within the theory applicable on the interval of scales $(\ell_{\text{UV}}, \ell_{\text{IR}})$ and within the theory applicable on the interval of scales $(\ell_{\text{UV}} + \delta\ell, \ell_{\text{IR}})$, then the results of these two computations necessarily agree on the intersection of these two intervals of scales. This last statement makes precise the sense in which all of the theories along a renormalization group trajectory describe the same physics.

In their second analysis ACGSJ seem to lose sight of this last fact. They first numerically measure the spectral dimension $d_s(\sigma)$ as a function of discrete diffusion time σ , and they then phenomenologically model the functional form of $d_s(\sigma)$ as

$$d_s^{(\text{fit})}(\sigma) = a - \frac{b}{c + \sigma} \quad (1)$$

for parameters a , b , and c determined by a best fit of $d_s^{(\text{fit})}(\sigma)$ to $d_s(\sigma)$. They apply this model to the same two sets of ensembles of causal triangulations, obtaining values of a , b , and c for each ensemble. Since the continuous diffusion time has dimensions of length squared, they next consider scaling σ , which is just a dimensionless counting parameter, as follows:

$$\sigma \longrightarrow \left(\frac{\mathbf{a}_{\text{ref}}}{\mathbf{a}}\right)^2 \sigma \quad (2)$$

in which \mathbf{a}_{ref} is the lattice spacing of the ensemble characterized by $\kappa_0 = 2.2$ and $\Delta = 0.6$, and \mathbf{a} is the lattice spacing of another ensemble. (They denote the ratio $\mathbf{a}_{\text{ref}}/\mathbf{a}$ as $1/\mathbf{a}_{\text{rel}}$.) They obtain values of $\mathbf{a}_{\text{ref}}/\mathbf{a}$ for each ensemble by maximizing the overlap of $d_s^{(\text{fit})}(\sigma)$ for all of the ensembles within both sets. Finally, they contend that the values of $\mathbf{a}_{\text{ref}}/\mathbf{a}$ so obtained indicate the relative change in the lattice spacing from the reference ensemble to each of the other ensembles.

For what reason do ACGSJ expect their determinations of $d_s^{(\text{fit})}(\sigma)$ for each ensemble to overlap with σ scaled as in equation (2)? I maintain that their unstated reasoning proceeds as follows. If the ensembles considered all fall along a renormalization group trajectory, then numerical measurements probe the same spectral dimension on different intervals of scales. The scaling in equation (2) compensates for the differences in these intervals' ultraviolet scales, resulting in the several measurements of the same spectral dimension overlapping. On the contrary, if the ensembles considered do not all fall along a renormalization group trajectory, then numerical measurements

probe different spectral dimensions on different intervals of scales. The scaling in equation (2) does not account for the differences in these spectral dimensions, resulting in the several measurements of the spectral dimension not overlapping.

In their determinations of $\mathbf{a}_{\text{ref}}/\mathbf{a}$, ACGSJ have therefore assumed that all of the ensembles considered fall along a renormalization group trajectory. They neither invoke this assumption nor demonstrate that these ensembles all fall along a renormalization group trajectory, so their comparison of values of $\mathbf{a}_{\text{ref}}/\mathbf{a}$ remains unjustified.

Cooperman discusses how one might compare numerical measurements of the spectral dimension across ensembles characterized by different bare couplings [19]. He argues that, for ensembles all falling along a renormalization group trajectory, their spectral dimensions are of the same shape if the diffusion time is appropriately scaled. Eyeballing ACGSJ's figure 4, which depicts $d_s^{(\text{fit})}(\sigma)$ scaled as in equation (2) for all of the ensembles considered, the several spectral dimensions do not all appear to be of the same shape. The scaling of σ in equation (2), based on its canonical scaling dimension, may not be correct for sufficiently small diffusion times on which the spectral dimension exhibits decidedly nonclassical behavior. Alternatively, all of the ensembles considered may not fall along a renormalization group trajectory, a possibility suggested by the conjectured renormalization group trajectory depicted in ACGSJ's figure 7. With these considerations in mind, ACGSJ must carefully examine the consistency of their comparison of numerical measurements of the spectral dimension for different values of the bare couplings.

If ACGSJ can justify the two assumptions elucidated above, then their analyses will make important contributions to the search for a continuum limit of causal dynamical triangulations. In the absence of justification, the import of their conclusions is seriously in doubt. I fear that to justify these assumptions they must tackle one of the most difficult problems facing the causal dynamical triangulations approach—and indeed all approaches—to the quantization of gravity: the construction of meaningful physical observables beyond the few already identified.

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